Noise Power Ratio (NPR)—A 65-Year Old Telephone System Specification Finds New Life in Modern Wireless Applications

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INTRODUCTION

The concept of Noise Power Ratio (NPR) has been around since the early days of frequency division multiplexed (FDM) telephone systems. The NPR is simply a measure of the "quietness" of an unused channel in a multi-channel system when there is random activity on the others. Noise and intermodulation distortion products fall into the unused channel causing less than ideal performance. Originally used to check 4-kHz wide voice channels in FDM links, the same concept is useful today in characterizing multichannel wideband communication systems—but there are some important differences in the modern measurement techniques.

HISTORY OF NPR

Noise power ratio testing has been used since the early days of Frequency Division Multiplexed (FDM) communication systems. In a typical FDM system, 4-kHz wide voice channels are "stacked" in frequency bins for transmission over coaxial, microwave, or satellite equipment. The number of channels depends on the system. A *group* is composed of 12 voice channels and occupies a bandwidth of 48 kHz. Similarly, a *supergroup* has 60 channels and occupies a bandwidth of 240 kHz, and a *mastergroup* has 300 channels and occupies a bandwidth of approximately 1.3 MHz. Supergroups and mastergroups are often combined to make up even higher capacity systems. For instance, an 1800-channel system occupies a bandwidth of approximately 8 MHz.

At the receiving end of the transmission link, the FDM data is demultiplexed and converted back to 4-kHz individual voiceband channels. The FDM signal is therefore composed of many individual voice channels and passes through amplifiers, repeaters, channel banks, etc., which add noise and distortion to the signal. Early studies at Bell Telephone Labs (Reference 1) led to the conclusion that the composite signal in an FDM system having more than approximately 100 channels can be approximated by Gaussian noise having a bandwidth equal to the bandwidth of the combined FDM signal. For instance, a 1800-channel FDM signal is approximated by Gaussian noise with a bandwidth of 8.2 MHz.

The "quality" of an individual voice channel is then measured by first assuming that there are random "talkers" on all channels except the specific 4-kHz channel under test. An individual 4-kHz channel can therefore be measured for "quietness" using a narrow-band notch (bandstop) filter and a specially tuned receiver which measures the noise power inside the 4-kHz notch as shown in Figure 1A.

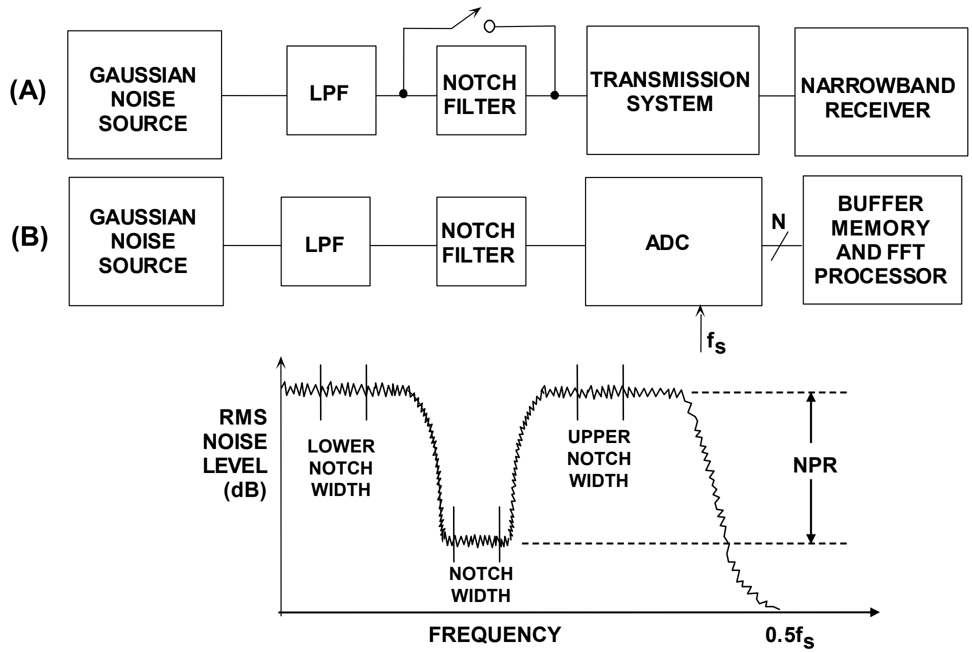

Figure 1: Noise Power Ratio (NPR) Measurements

Noise Power Ratio (NPR) measurements are straightforward in an analog transmission system (Figure 1A). With the notch filter out, the rms noise power of the signal inside the notch is measured by the narrowband receiver. The notch filter is then switched in, and the residual noise inside the notch is measured. The ratio of these two readings expressed in dB is the NPR. Several notch frequencies across the noise bandwidth (low, midband, and high) are tested to characterize the system adequately. Details of early NPR test equipment and the measurements can be found in Reference 4. NPR measurements on ADCs are made in a similar manner, except the analog receiver is replaced by a buffer memory and an FFT processor which performs the calculations as shown in Figure 1B. There are some cases where the combined FDM signal is converted to digital with an ADC, transmitted, and then converted back to analog using a DAC at the receiver. In this case, the analog method shown in Figure 1A is utilized in performing the NPR test.

In a 1939 article (Reference 1), Holbrook and Dixon performed an analysis of FDM systems in an effort to determine the optimum channel "loading" levels. Their work led to the fundamental theory of multichannel noise loading. The goal is to set the signal level (or "loading") to a value which will give the highest NPR. The NPR is plotted as a function of rms noise level referred to the peak range of the system. For very low noise loading levels, the undesired noise (in non-digital systems) is primarily thermal noise and is independent of the input noise level. Over this region of the curve, a 1-dB increase in noise loading level causes a 1-dB increase in NPR. As the noise loading level is increased, the amplifiers and repeaters in the system begin to overload, creating intermodulation products which cause the noise floor of the system to increase. As the input noise continues to increase, the effects of "overload" noise predominate, and the NPR is reduced dramatically. FDM systems are usually operated at a noise loading level a few dB below the point of maximum NPR to allow headroom for peak busy hours.

System NPR recommendations for FDM systems were formalized in 1966 by the CCITT/CCIR to measure the transmission characteristics of Frequency Division Multiple (FDM) communications links (see Reference 4).

In a digital system containing an ADC, the noise within the notch is primarily quantization noise when low levels of input noise are applied. However, for very low amplitude signals (less than 1-LSB peak-to-peak), the resulting noise reverts to the input-referred noise of the ADC. For signals that exercise several LSBs of the ADC, the NPR curve is linear, and quantization noise predominates. As the noise level increases, there is a one-for-one correspondence between the noise level and the NPR. At some level, however, "clipping" noise caused by the hard-limiting action of the ADC begins to dominate. The ADC hard-limiting "clipping" noise is somewhat different from the soft-limiting "overload" noise of an analog FDM and results in a "steeper" downward slope in the clipping region.

THEORETICAL NPR FOR A DIGITAL SYSTEM

Several papers have been written over the years deriving the theoretical NPR of an ideal n-bit ADC (for example, References 5, 6, and 7). Reference 6 is the most complete, and shows the derivation for both uniformly distributed noise and Gaussian noise. However, Gaussian noise is much more relevant to NPR testing. The derivation is not difficult but does involve some partial integration. Since the "clipping" noise component not have a closed-form solution, numerical methods must be used to actually compute the theoretical NPR numbers.

A theoretical curve for 10, 12, 14, and 16-bit ADCs is shown in Figure 2. Understanding the definitions of the terms V_O, σ, k, and the rms loading level ($-20\log_{10}k$) are very important in order to avoid confusion.

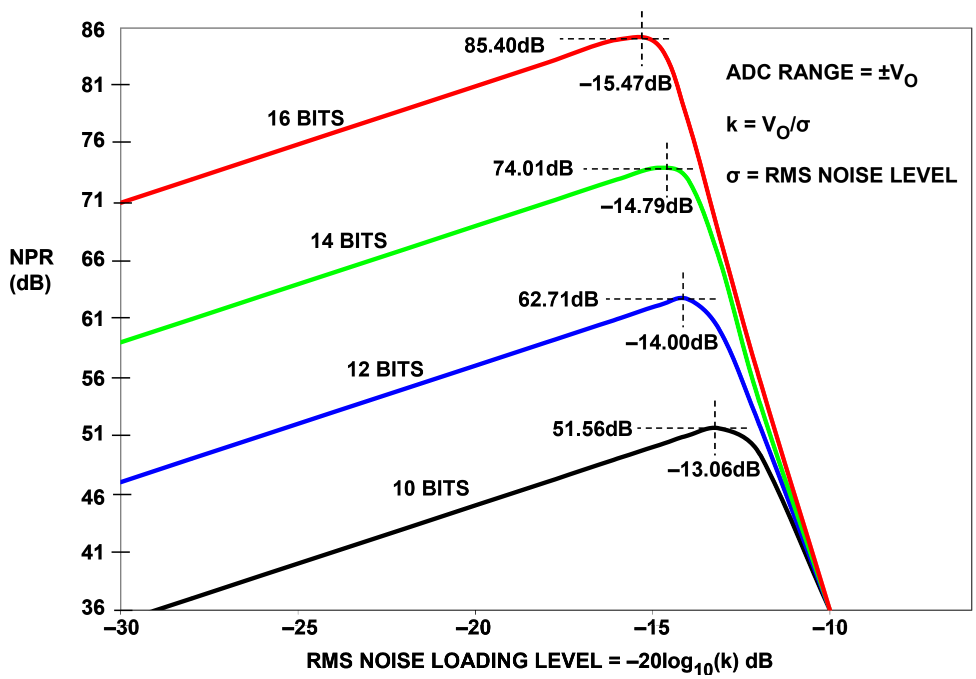

Figure 2: Theoretical NPR for 10, 12, 14, and 16-bit ADCs

It is important to understand that these curves are based on an ideal ADC where the only noise is the quantization noise and the clipping noise. In practice, the actual level of performance will be less than theoretical, depending upon the particular ADC under test.

The ADC input range is bipolar, and is $\pm V_O$ full-scale (hence $2V_O$ peak-to-peak). The input rms noise level is σ, and the noise-loading factor k (also called the *crest factor*) is defined as V_O/σ. The value of k is therefore the *peak-signal-to-rms-noise-ratio*, where k is expressed as a numerical ratio. Again, it is important to note that a *peak signal* of V_O implies a peak-to-peak full-scale input of $2V_O$. This can become a point of confusion. Another way to put it is that a full-scale sinewave given by $v(t) = V_O \cdot \sin\omega t$ exactly fills the ADC input range. This is why V_O is referred to as the *peak amplitude*.

The reciprocal of k is the *rms-noise-to-peak-signal-ratio*, and the rms noise loading level is defined as 1/k expressed in dB:

$$\text{RMS Noise Loading Level} = 20\log_{10}\left(\frac{1}{k}\right) = -20\log(k). \qquad \text{Eq. 1}$$

The derivation for theoretical NPR can be broken into two parts. The first part derives the theoretical quantization noise power of an ideal n-bit ADC. The second part derives the "clipping noise" power due to the limiting action of the ADC. The total noise power is the sum of the two noise powers. The complete error waveform showing the two regions is shown in Figure 3.

The theory is based on several assumptions. First, the quantization error signal is not correlated to the input signal. This is valid provided the signal amplitude is at least several LSBs in amplitude and the resolution of the ADC is at least 6-bits. Second, the sampling rate is twice the input noise bandwidth. Third, the ADC acts as an ideal limiter for out-of-range signals. These three assumptions are valid for most practical systems and lead to a relatively straightforward solution.

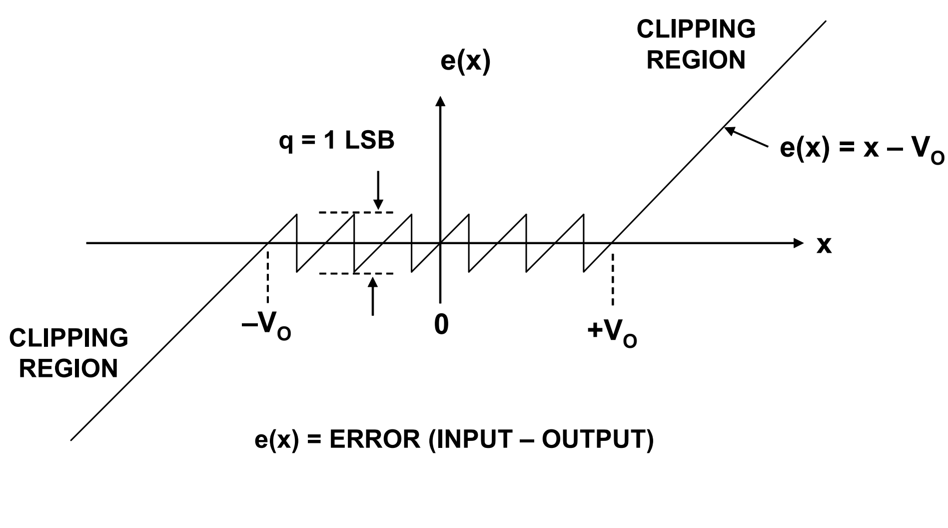

Figure 3: Ideal ADC Error Waveform

The quantization noise component (expressed as the square of the actual quantization noise voltage to yield noise power), has been shown to be (see Reference 2, for example):

$$N_Q = \frac{q^2}{12}, \qquad \text{Eq. 2}$$

where q is the weight of the least significant bit (LSB). It should be noted that this is the quantization noise power measured over the full Nyquist bandwidth dc to $f_s/2$. If the signal bandwidth is reduced, the noise in the reduced bandwidth is proportionally less, and a correction factor must be added (discussed later in this paper).

Continuing with the derivation we know that, $q = 2V_O/2^n$. Therefore, from Equation 2:

$$N_Q = \frac{q^2}{12} = \frac{\left(2V_O/2^n\right)^2}{12} = \frac{V_O^2}{3 \cdot 2^{2n}}. \qquad \text{Eq. 3}$$

However, $k = V_O/\sigma$, therefore $V_O = k\sigma$, and substituting for V_O in Equation 3 yields:

$$N_Q = \frac{k^2\sigma^2}{3 \cdot 2^{2n}}. \qquad \text{Eq. 4}$$

Now, refer to Figure 3 for the derivation of the clipping noise power, N_C.

The clipping noise power is given by the following general equation:

$$N_C = \int_{-\infty}^{+\infty} e^2(x)P(x)dx \qquad \text{Eq. 5}$$

From Figure 3B,

$$e(x) = x - V_O, \quad \text{for } x > V_O \text{ , and} \quad \text{therefore} \qquad \text{Eq. 6}$$

$$N_C = 2\int_{V_O}^{\infty} (x - V_O)^2 P(x)dx, \qquad \text{Eq. 7}$$

where P(x) is the Gaussian probability density function and is given by:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}. \qquad \text{Eq. 8}$$

Substituting $V_O = k\sigma$, and combining Equation 8 with Equation 7 yields:

$$N_C = 2\int_{k\sigma}^{\infty} (x - k\sigma)^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} dx \qquad \text{Eq. 9}$$

The final results of the integration (see Appendix for complete derivation) yields:

$$N_C = 2\sigma^2(k^2+1)[1 - N(k)] \;\; - k\sigma^2\sqrt{\frac{2}{\pi}}\, e^{-k^2/2} \qquad \text{Eq. 10}$$

Where N(k) is the Normal Distribution Function:

$$N(k) = \int_{-\infty}^{k} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt\,. \qquad \text{Eq. 11}$$

For calculation purposes, the function [1 – N(k)] can be approximated by the following expression:

$$1 - N(k) \approx \frac{1}{k\sqrt{2\pi}} e^{-k^2/2}\left[1 - \frac{1}{k^2+2} + \frac{1}{(k^2+2)(k^2+4)} - \frac{5}{(k^2+2)(k^2+4)(k^2+6)} + \frac{9}{(k^2+2)(k^2+4)(k^2+6)(k^2+8)} - \frac{129}{(k^2+2)(k^2+4)(k^2+6)(k^2+8)(k^2+10)}\right]. \qquad \text{Eq. 12}$$

The total noise, N_T, can now be calculated by adding Equation 4 and Equation 10:

$$N_T = N_Q + N_C = \frac{k^2\sigma^2}{3\cdot 2^{2n}} + 2\sigma^2(k^2+1)[1 - N(k)] \;\; - k\sigma^2\sqrt{\frac{2}{\pi}}\, e^{-k^2/2}, \qquad \text{Eq. 13}$$

$$\frac{N_T}{\sigma^2} = \frac{k^2}{3\cdot 2^{2n}} + 2(k^2+1)[1 - N(k)] \;\; - k\sqrt{\frac{2}{\pi}}\, e^{-k^2/2}. \qquad \text{Eq. 14}$$

$$NPR = 10\log\left(\frac{\sigma^2}{N_T}\right) = -10\log\left(\frac{N_T}{\sigma^2}\right) \qquad \text{Eq. 15}$$

Figure 4 shows the theoretical peak value of NPR and the corresponding value of k for ADCs having resolutions between 8 and 20 bits. The vertical axis is NPR (expressed in dB per Equation 15). The horizontal axis is the Gaussian noise loading level with respect to the peak signal level, σ/V_O, expressed in dB.

BITS	k OPTIMUM	k(dB)	MAX NPR (dB)
8	3.92	11.87	40.60
9	4.22	12.50	46.05
10	4.50	13.06	51.56
11	4.76	13.55	57.12
12	5.01	14.00	62.71
13	5.26	14.41	68.35
14	5.49	14.79	74.01
15	5.72	15.15	79.70
16	5.94	15.47	85.40
18	6.34	16.04	96.88
20	6.78	16.62	108.41

ADC Range = $\pm V_O$
$k = V_O / \sigma$
σ = RMS Noise Level

Figure 4: Theoretical Maximum NPR for 8 to 20-bit ADCs

Again it is important to remember that this is the NPR obtained when the input signal noise occupies the full Nyquist bandwidth, dc to $f_s/2$. For the case of oversampling, where the signal bandwidth, BW, is less than $f_s/2$, the correction factor of $10\log_{10}[f_s/(2\cdot BW)]$, often referred to as *process gain*, must be added to the NPR given in Equation 15:

$$NPR = 10\log\left(\frac{\sigma^2}{N_T}\right) + 10\log\left(\frac{f_s}{2\cdot BW}\right). \qquad \text{Eq. 16}$$

In multi-channel high frequency communication systems, where there is little or no phase correlation between channels, NPR can be used to measure the distortion and noise caused by a large number of individual channels, similar to an FDM system. A notch filter is placed between the noise source and the ADC, and an FFT output is used in place of an analog receiver. The width of the notch filter is set for about 500 kHz to 2 MHz as shown in Figure 5 for the AD9229 12-bit 65-MSPS ADC. The sampling rate is 65 MSPS, the notch is centered at 18 MHz, and the NPR is the "depth" of the notch. An ideal ADC will only generate the theoretical value of quantization noise, however a practical one has additional noise components due to additional noise and intermodulation distortion caused by ADC imperfections. Notice that the NPR is about 60.8 dB compared to 62.7-dB theoretical.

Making NPR measurements digitally requires that the FFT have a sufficient number of samples such that there are at least 25 to 50 samples within the filter notch. There are obviously tradeoffs between the width of the notch and the FFT size. However, the notch width should not be wider than about 10% of the noise bandwidth, or the test results may not be valid.

In the example shown in Figure 5 for the AD9229, the FFT size was 16,384 which gives a frequency resolution of 65 MSPS/16,384 = 3.97 kHz. Since the notch filter width is approximately 1 MHz at the bottom of the notch, approximately 250 samples fall within the notch. Due to the specific requirements regarding the center frequency, width, and band-stop rejection, custom-made notch filters are generally required in order to implement NPR tests on ADCs. Achieving good results is difficult using just a simple filter and wideband noise source. Wideband Gaussian noise generators, such as the NoiseCom DNG7500, are available that allow the user to custom shape the noise according to their application. Using a combination of a Gaussian noise shaping generator and notch filter makes this test easier to implement. The results of several FFTs must then be averaged in order to reduce the variation in the NPR results from run to run since there are only a limited number of samples which fall inside the notch itself. The data shown in Figure 5 represents the average of the NPR results for 5 individual FFT runs.

NPR should be measured at several different frequencies across the noise bandwidth, thereby requiring several notch filters. Some degradation will occur at the higher frequencies—very similar to the degradation in other ADC ac specifications such as SNR and SFDR.

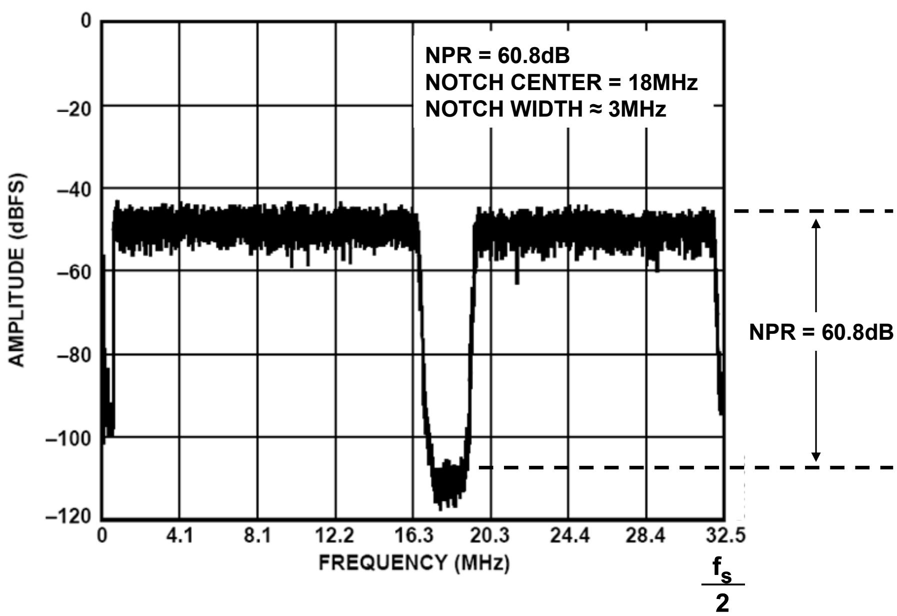

Figure 5: AD9229 12-bit, 65-MSPS ADC NPR Measures 60.8 dB (62.7 dB Theoretical)

SUMMARY

We have shown how NPR is used in standard FDM systems to characterize the noise and intermodulation distortion of multi-channel system where the voice channel width is 4 kHz. It can also be used to determine the optimum signal level to give maximum dynamic range. This 65-year old concept is still useful today in modern multichannel wireless systems. Bandwidths and channel spacings are higher, but the same concepts still apply. In many cases, NPR is a good approximation to complicated multi-tone testing and embodies the specific features of many applications when testing your system's dynamic range (Reference 7).

Although the single tone or two-tone sinewave signal is by far the most popular method for testing ADCs for wideband applications, NPR testing offers a relatively easy method using a Gaussian noise input to simulate a broadband multitone signal without the need for generating a large number of single tone sinewaves.

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APPENDIX

In this appendix, we will show how to evaluate the following integral from Equation 9.

$$N_C = 2\int_{k\sigma}^{\infty}(x-k\sigma)^2 \cdot \frac{1}{\sigma\sqrt{2\pi}}\, e^{-x^2/2\sigma^2}\, dx\,. \qquad \text{Eq. A1}$$

This integral is of the form:

$$N_C = C\int_{A}^{\infty}(x-A)^2\, e^{-Bx^2}\, dx \qquad \text{Eq. A2}$$

where

$$A = k\sigma\,, \qquad B = \frac{1}{2\sigma^2}\,, \qquad C = \frac{\sqrt{2}}{\sigma\sqrt{\pi}}\,. \qquad \text{Eq. A3}$$

Integrating Eq. A2:

$$N_C = C\int_{A}^{\infty}(x-A)^2\, e^{-Bx^2}\, dx = C\int_{A}^{\infty}x^2 e^{-Bx^2}\, dx - 2AC\int_{A}^{\infty}x e^{-Bx^2}\, dx + CA^2\int_{A}^{\infty}e^{-Bx^2}\, dx$$

Eq. A4

Now, evaluate the first integral using partial integration:

$$\int_{A}^{\infty}x^2 e^{-Bx^2}\, dx = -\frac{1}{2B}\int_{A}^{\infty}x(-2Bx)\, e^{-Bx^2}\, dx\,. \qquad \text{Eq. A5}$$

The fundamental equation for partial integration is:

$$\int u\,dv = uv - \int v\,du\,. \qquad \text{Eq. A6}$$

Let $u = x$ and $dv = -2Bxe^{-Bx^2}\, dx$.

Then

$$\int_A^{\infty} x^2 e^{-Bx^2} dx = -\frac{1}{2B}\left\{ xe^{-Bx^2}\Big|_A^{\infty} - \int_A^{\infty} e^{-Bx^2} dx \right\} \qquad \text{Eq. A7}$$

$$= \frac{A}{2B} e^{-BA^2} + \frac{1}{2B}\int_A^{\infty} e^{-Bx^2} dx \qquad \text{Eq. A8}$$

Evaluating the second integral in Equation A4:

$$\int_A^{\infty} xe^{-Bx^2} dx = -\frac{1}{2B}\int_A^{\infty} (-2Bx) e^{-Bx^2} dx = -\frac{1}{2B} e^{-Bx^2}\Big|_A^{\infty} = \frac{1}{2B} e^{-BA^2} \qquad \text{Eq. A9}$$

Substituting Equation A8 and A9 into Equation A4:

$$N_C = C\int_A^{\infty} (x-A)^2 e^{-Bx^2} dx$$

$$= \frac{CA}{2B} e^{-BA^2} + \frac{C}{2B}\int_A^{\infty} e^{-Bx^2} dx - \frac{2AC}{2B} e^{-BA^2} + A^2 C\int_A^{\infty} e^{-Bx^2} dx$$

$$= \left(\frac{AC}{2B} - \frac{2AC}{2B}\right) e^{-BA^2} + \left(\frac{C}{2B} + A^2 C\right)\int_A^{\infty} e^{-Bx^2} dx$$

$$= -\frac{AC}{2B} e^{-BA^2} + C\left(\frac{1}{2B} + A^2\right)\int_A^{\infty} e^{-Bx^2} dx \qquad \text{Eq. A10}$$

Now from Eq. A3, substituting $A = k\sigma$, $B = \frac{1}{2\sigma^2}$, and $C = \frac{\sqrt{2}}{\sigma\sqrt{\pi}}$ into Eq. A10:

$$N_C = -k\sigma^2\sqrt{\frac{2}{\pi}}\, e^{-k^2/2} + \frac{2\sigma}{\sqrt{2\pi}}\left(1+k^2\right)\int_{k\sigma}^{\infty} e^{-x^2/2\sigma^2} dx \qquad \text{Eq. A11}$$

$$\text{Define} \quad t = \frac{x}{\sigma}, \quad x = t\,\sigma, \quad dx = \sigma\, dt \qquad \text{Eq. A12}$$

Then substituting into Equation A11 and rearranging yields:

$$N_C = 2\sigma^2\left(1+k^2\right)\left[1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{k} e^{-t^2/2}dt\right] - k\sigma^2\sqrt{\frac{2}{\pi}}\; e^{-k^2/2} \qquad \text{Eq. A13}$$

$$N_C = 2\sigma^2(1+k^2)[1-N(k)] \quad - k\sigma^2\sqrt{\frac{2}{\pi}}\; e^{-k^2/2} \qquad \text{Eq. A14}$$

Where $N(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{k} e^{-t^2/2}dt$, the Normal Distribution Function Eq. A15